

Sister Nibedita Government General Degree College for Girls
Department of Mathematics

Multiple Choice Questions : Select (a), (b), (c) or (d), whichever is correct.

1. An infinite subset of enumerable set is
 - a) countable
 - b) uncountable
 - c) finite
 - d) none of the above
2. Derived set of (a,b) is
 - a) (a,b]
 - b) Φ
 - c) [a,b)
 - d) [a,b]
3. $\mathbb{R}-\mathbb{Z}$ is
 - a) open set
 - b) closed set
 - c) clopen set
 - d) none of the above
4. $I_n = [0, 1/n]$, where n is natural number. Then $\bigcap_{n=1}^{\infty} I_n$ is
 - a) (0,1/n)
 - b) (0,1)
 - c) Φ
 - d) {0}
5. If $\{a_n\}$ is monotone increasing bounded sequence. Then $\{a_n\}$ is
 - a) always convergent
 - b) always divergent
 - c) oscillates finitely
 - d) none of the above
6. $\lim_{n \rightarrow \infty} \frac{(n!)^{\frac{1}{n}}}{n}$ equals to
 - a) e
 - b) 1/e
 - c) 1
 - d) 0
7. Sum of two Cauchy sequence is
 - a) Cauchy
 - b) not necessarily Cauchy
 - c) never Cauchy
 - d) bounded not necessarily
8. The limit of the sequence $\{x_n\}$, where $x_{n+1} = \sqrt{2 + x_n}$, $x_1 = \sqrt{2}$
 - a) $\sqrt{2}$
 - b) 2
 - c) 1
 - d) -1

Long Answer Type Questions:

1. Prove that a sequence cannot converge in more than one limit.
2. State and prove Cauchy Convergence criteria.
3. Let S be any non empty subset of R that is bounded above and let a be any real number then show that $\text{Sup}(a+S) = a + \text{Sup}S$

4. Show that there exists a positive real number x such that $x^2=2$.
5. State completeness property of \mathbb{R}
6. Let $\{x_n\}$ converges to x and $\{y_n\}$ be a sequence of non zero real numbers that converges to y and if $y \neq 0$, then show that $\{x_n/y_n\}$ converges to x/y .
7. Show that a convergent sequence is bounded. Is the converse true?
8. Find the limit of the sequence $\{\sin n/n\}$.
9. State and prove Archimedean property of \mathbb{R} .
10. Check whether the following sequences are Cauchy or not.
 - a) $a_n=1+1/2!+1/3!+\dots+1/n!$
 - b) $a_n=1+1/2+1/3+\dots+1/n$
11. Using subsequence show that the sequence $\{\cos n\}$ is not convergent.
12. Let $a_n=1/(n+1)^2+1/(n+2)^2+\dots+1/(n+n)^2$. Show that $\{a_n\}$ is monotone bounded and converges to 0.
13. Let $K=\{s+t^{1/2}: s,t \in \mathbb{Q}\}$. Show that K satisfies the following:
 - a) $x,y \in K$ then $x+y \in K$ and $xy \in K$.
 - b) for any non zero x and $x \in K$ then $1/x \in K$.
14. Show that union of infinite number of open set is open? Is it true for closed set?
15. Find the limit point of the set $\{1/m+1/n: m,n \in \mathbb{N}\}$.
16. Show that every infinite bounded set of real numbers has a limit point.
17. Intersection of two nbd of a point is nbd of that point.
18. Show that the set $S=\{x \in \mathbb{R} : 0 < x < 1\}$ is open but not closed.