

**Sister Nibedita Government General Degree College for Girls**  
**Department of Mathematics**

**Multiple Choice Questions : Select (a), (b), (c) or (d), whichever is correct.**

1. An infinite subset of enumerable set is
  - a) countable
  - b) uncountable
  - c) finite
  - d) none of the above
  
2. Derived set of (a,b) is
  - a) (a,b]
  - b)  $\Phi$
  - c) [a,b)
  - d) [a,b]
  
3.  $\mathbb{R}-\mathbb{Z}$  is
  - a) open set
  - b) closed set
  - c) clopen set
  - d) none of the above
  
4.  $I_n = [0, 1/n]$ , where n is natural number. Then  $\bigcap_{n=1}^{\infty} I_n$  is
  - a) (0,1/n)
  - b) (0,1)
  - c)  $\Phi$
  - d) {0}
  
5. If  $\{a_n\}$  is monotone increasing bounded sequence. Then  $\{a_n\}$  is
  - a) always convergent
  - b) always divergent
  - c) oscillates finitely
  - d) none of the above
  
6.  $\lim_{n \rightarrow \infty} \frac{(n!)^{\frac{1}{n}}}{n}$  equals to
  - a) e
  - b) 1/e
  - c) 1
  - d) 0
  
7. Sum of two Cauchy sequence is
  - a) Cauchy
  - b) not necessarily Cauchy
  - c) never Cauchy
  - d) bounded not necessarily
  
8. The limit of the sequence  $\{x_n\}$ , where  $x_{n+1} = \sqrt{2 + x_n}$ ,  $x_1 = \sqrt{2}$ 
  - a)  $\sqrt{2}$
  - b) 2
  - c) 1
  - d) -1

**Long Answer Type Questions:**

1. Prove that a sequence cannot converge in more than one limit.
2. State and prove Cauchy Convergence criteria.
3. Let S be any non empty subset of R that is bounded above and let a be any real number then show that  $\text{Sup}(a+S) = a + \text{Sup}S$

4. Show that there exists a positive real number  $x$  such that  $x^2=2$ .
5. State completeness property of  $\mathbb{R}$
6. Let  $\{x_n\}$  converges to  $x$  and  $\{y_n\}$  be a sequence of non zero real numbers that converges to  $y$  and if  $y \neq 0$ , then show that  $\{x_n/y_n\}$  converges to  $x/y$ .
7. Show that a convergent sequence is bounded. Is the converse true?
8. Find the limit of the sequence  $\{\sin n/n\}$ .
9. State and prove Archimedean property of  $\mathbb{R}$ .
10. Check whether the following sequences are Cauchy or not.
  - a)  $a_n=1+1/2!+1/3!+\dots+1/n!$
  - b)  $a_n=1+1/2+1/3+\dots+1/n$
11. Using subsequence show that the sequence  $\{\cos n\}$  is not convergent.
12. Let  $a_n=1/(n+1)^2+1/(n+2)^2+\dots+1/(n+n)^2$ . Show that  $\{a_n\}$  is monotone bounded and converges to 0.
13. Let  $K=\{s+t^{1/2}: s,t \in \mathbb{Q}\}$ . Show that  $K$  satisfies the following:
  - a)  $x,y \in K$  then  $x+y \in K$  and  $xy \in K$ .
  - b) for any non zero  $x$  and  $x \in K$  then  $1/x \in K$ .
14. Show that union of infinite number of open set is open? Is it true for closed set?
15. Find the limit point of the set  $\{1/m+1/n: m,n \in \mathbb{N}\}$ .
16. Show that every infinite bounded set of real numbers has a limit point.
17. Intersection of two nbd of a point is nbd of that point.
18. Show that the set  $S=\{x \in \mathbb{R} : 0 < x < 1\}$  is open but not closed.